Habits and Multiple Equilibria

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Abstract

Abstract TBA.

A Introduction

Brief introduction TBA

B A Structural Model of Taste Changes

Several structural models can give rise to the persistent long-run effects of public policies we identified in the main paper. In this section we propose one particular structural model of taste changes under which even temporary policy interventions can lead to persistent effects in the long run. This basic model is consistent with the consumption patterns documented in the paper. The model extends the habit formation model by Becker and Murphy (1988) to allow for two habit-forming goods, illustrating that in this situation several steady-state consumption patterns are possible even in the absence of any unobserved individual heterogeneity. A person's consumption shares in steady state depend solely on his initial consumption pattern. Moreover, it is hard to change these consumption patterns even with very large shocks once the stock of habit is sufficiently large. Hence, policies aimed at increasing the relative price of one good may not induce everybody or even many to reduce the consumption of this good. Instead, due to the stock of habits already accumulated, people who are accustomed to this particular good will still prefer it even after the policy change. This implies that policies that influence the initial choices of younger generations can have long-run consequences over their entire life span—intended or otherwise.

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B.1 Model Setup

For simplicity we assume that consumers spend all of their budget on two habit-forming goods, beer and vodka. We also assume that consumers are myopic, i.e., that they maximize only current utility and do not save, that there are no outside goods, that income does not change over time, and that there is no uncertainty.[1]

The individual derives flow utility $u(v_t, b_t, H_t^v, H_t^b)$ from consuming vodka v_t and beer b_t and also from the corresponding stocks of habit H_t^v and H_t^b. The utility function has properties that are common in the literature, specifically that $u_g > 0$, $u_{gg} < 0$, and $u_{gH_g} > 0$ with $g \in \{b, v\}$. These assumptions imply in particular that the marginal utilities of consuming beer or vodka are positive and increasing with the stock of habit of the corresponding good. Assuming a common rate of depreciation δ of the two habit stocks, they evolve as

$$H_{t+1}^g = (1-\delta)H_t^g + g_t,\ H_0^g \geq 0,\ \delta \in [0,1]. \tag{1}$$

The budget constraint is $p_{v_t} v_t + b_t = y_t$. Without loss of generality, we focus on interior solutions.[2] The first-order condition of this optimization problem is

$$u_v(v_t, y_t - p_{v_t}v_t, H_t^v, H_t^b) - p_{v_t} u_b(v_t, y_t - p_{v_t}v_t, H_t^v, H_t^b) = 0\ , \tag{2}$$

where u_v and u_b are the partial derivatives with respect to the first and second arguments, respectively. Since we are interested in the long-run effects of habit formation, we focus our analysis on the properties of the model's steady state. In the steady state where prices, income, and consumption are constant such that $p_{v_t} = p_v$, $y_t = y$, and $g_t = g$, the expression for the stocks of habit is g/δ. The first-order condition that implicitly defines the steady state can then be rewritten as

$$u_v(v,\ y - p_v v,\ v/\delta,\ (y-p_v v)/\delta) - p_v u_b(v,\ y - p_v v,\ v/\delta,\ (y - p_v v)/\delta) = 0\ . \tag{3}$$

In general, this is a non-monotonic function in the steady-state vodka consumption v.[3] Depending on the parametrization of the utility function u, equation (3) may have a different number of solutions. Figure A.6 illustrates that for certain parametrizations, there is a unique solution, but for many other parametrizations several steady states exist, up to a continuum of solutions.[4] These multiple equilibria are derived without any consumer heterogeneity except

[1]Below we reach the same qualitative conclusions if consumers are forward looking and solve a fully dynamic problem.

[2]If there are corner solutions, there is always a symmetric specification with at least 3 equilibria where the two stable equilibria have a consumption share in each good of either 1 or 0.

[3]This condition can also be expressed as a function of the share of vodka, $S^v = \frac{v}{v+b}$, by using the fact that $v = \frac{y \cdot S^v}{1-(1-p_v)S^v}$; see below.

[4]See below for a proof. Similar results are obtained for the model with forward-looking consumers because the steady-state Euler equation is also non-monotonic in the consumption levels.

for differences in initial conditions. A person who initially consumes primarily beer will also prefer beer in the long-run steady state, and vice versa for vodka.

B.2 Model Properties and Extensions

This section shows that the model above with two habit forming goods can have any number of equilibira. We then provide three numerical examples that generate, respectively, one, three, and an infinite number of equilibria. We also show how to map the steady state, which the model expresses in levels, to alcohol shares, which is the concept we use in our empirical analysis. Finally, we show that these insights from be basic myopic model extend to a model with forward-looking consumers.

B.2.1 Number of Equilibria in the Model with Myopic Consumers

The steady state first-order condition (FOC) for myopic agents as a function of the level of vodka consumption, v, is

$$\begin{aligned} F &= u_v(v,\ y-p_v v,\ [\delta/(1-\delta)]v,\ [\delta/(1-\delta)][y-p_v v]) \\ &\quad -p_v u_b(v,\ y-p_v v,\ [\delta/(1-\delta)]v,\ [\delta/(1-\delta)][y-p_v v]) = 0. \end{aligned}$$

Differentiating F with respect to v yields

$$u_{vv}-p_v u_{vb}+\delta/(1-\delta)u_{vH^v}-p_v\delta/(1-\delta)u_{vH^b}-p_v[u_{bv}-p_v u_{bb}+\delta/(1-\delta)u_{bH^v}-p_v\delta/(1-\delta)u_{bH^b}].$$

Given the assumptions that $u_{gg}<0$, $u_{H^gH^g}<0$, and $u_{gH^g}>0$, some terms in this expression are positive, e.g., $\delta/(1-\delta)u_{vH^v}$, $p_v^2\delta/(1-\delta)u_{bH^b}$, and some are negative, e.g., u_{vv}, $p_v^2 u_{bb}$. Therefore, the sign of the overall sum is ambiguous.

B.2.2 Numerical Examples

One Equilibrium Let the utility function be $u = ln(b)\cdot L_b + ln(v)\cdot L_v$—with $L_g = \ln(1.1+H^g)$ for $g\in\{b,v\}$—so that the marginal utility is $u_g = \frac{L_g}{g}$. The FOC is

$$\begin{aligned} 0 &= u_v - p_v\cdot u_b \\ &= \frac{L_v}{v} - \frac{p_v L_b}{b} \\ &= \frac{L_v}{p_v v} - \frac{L_b}{b} \\ &= \frac{L_v}{p_v v} - \frac{L_b}{y-p_v v}. \end{aligned}$$

Solving for v we obtain

$$v = \frac{L_v}{L_v + L_b} \cdot \frac{y}{p_v}.$$

Three Equilibria Let the utility function be $u = \sqrt{b} \cdot L_b + \sqrt{v} \cdot L_v$—with $L_g = \ln(1.1 + H^g)$ for $g \in \{b, v\}$—so that the marginal utility is $u_x = \frac{L_g}{2\sqrt{g}}$. Solving for v we obtain

$$v = \frac{R \cdot y}{1 + R \cdot p_v},$$

with $R = \left(\frac{L_v}{p_v \cdot L_b}\right)^2$.

Continuum of Equilibria Let the utility function be $u = \sqrt{b \cdot H^b} + \sqrt{v \cdot H^v}$, so that the marginal utility is $u_g = \frac{\sqrt{H^g}}{2\sqrt{g}}$. Solving for v we obtain

$$v = \frac{R \cdot y}{1 + R \cdot p_v},$$

with $R = \frac{H^v}{p_v^2 \cdot H^b}$.

B.2.3 Expressing the Model Solutions in Terms of Shares

$S_g = \frac{g}{b+v}$, $S_b + S_v = 1$, $p_v v + b = y$, and $\frac{S_v}{S_b} = \frac{v}{b}$. Hence,

$$\begin{aligned} v &= \frac{S_v}{S_b} b = \frac{S_v}{1 - S_v}(y - p_v v) \\ &= \frac{y \cdot S_v}{1 - (1 - p_v) S_v}. \end{aligned}$$

B.2.4 Allowing for Forward-Looking Consumers

We now relax the assumption of myopic behavior. Forward looking agents maximize the present value of utility from consuming beer and vodka, $U = u(v_t, b_t, H_t^v, H_t^b) + \sum_{i=1}^{\infty} \beta^i [u(v_{t+i}, b_{t+i}, H_{t+i}^v, H_{t+i}^b)]$. To keep the model simple, we follow Gruber and Köszegi (2001) and assume no savings and that the stock of habits evolves as follows:

$$H_{t+1}^g = \delta(H_t^g + g_t).$$

The FOC for v_t, after substituting for b_t using the budget constraints, is

$$u_{v_t} - p_{v_t} u_{b_t} + \sum_{i=1}^{\infty} \beta^i \delta^i (u_{H^v_{t+i}} - p_{v_t} u_{H^b_{t+i}}) = 0.$$

The FOC for v_{t+1} is

$$u_{v_{t+1}} - p_{v_{t+1}} u_{b_{t+1}} + \sum_{i=1}^{\infty} \beta^i \delta^i (u_{H^v_{t+i+1}} - p_{v_{t+1}} u_{H^b_{t+i+1}}) = 0.$$

Combining the two FOCs and analyzing the steady state we obtain the following Euler equation:

$$\begin{aligned} 0 = \quad & u_v(v, y - p_v v, \tfrac{\delta}{1-\delta} v, \tfrac{\delta}{1-\delta}[y - p_v v]) - p_v u_b(v, y - p_v v, \tfrac{\delta}{1-\delta} v, \tfrac{\delta}{1-\delta}[y - p_v v]) \\ & + \tfrac{\beta\delta}{1-\beta\delta}[u_{H^v}(v, y - p_v v, \tfrac{\delta}{1-\delta} v, \tfrac{\delta}{1-\delta}[y - p_v v]) - p_v u_{H^b}((v, y - p_v v, \tfrac{\delta}{1-\delta} v, \tfrac{\delta}{1-\delta}[y - p_v v])]. \end{aligned}$$

Assuming that $u_g \to \infty \ as \ g \to 0$ guarantees the existence of a steady state.

To check the possibility of multiple steady states, we can analyze the monotonicity of the right-hand side of the steady-state Euler equation by taking the first derivative with respect to v,

$$\begin{aligned} dRHS(v)/dv = \quad & u_{vv} - 2p_v u_{vb} + p_v^2 u_{bb} + \tfrac{\delta}{1-\delta}[u_{vH^v} - 2p_v u_{vH^b} + p_v^2 u_{bH^b}] \\ & + \tfrac{\beta\delta}{1-\beta\delta}[u_{vH^v} - p_v u_{bH^v} - p_v u_{vH^v} + p_v^2 u_{bH^b} + \tfrac{\delta}{1-\delta}[u_{H^v H^v} - 2p_v u_{H^v H^b} + p_v^2 u_{H^b H^b}]]. \end{aligned}$$

This expression can be both negative and positive. To see this, assume that the utility function is separable in the two goods and their stocks of habit. Then the expression above can be rewritten as

$$\begin{aligned} dRHS(v)/dv \ &= \left[u_{vv} + p_v^2 u_{bb} + \tfrac{\beta\delta}{1-\beta\delta}\tfrac{\delta}{1-\delta}(u_{H^v H^v} + p_v^2 u_{H^b H^b})\right] \\ & \quad + \left[(\tfrac{\delta}{1-\delta} + \tfrac{\beta\delta}{1-\beta\delta})(u_{vH^v} + p_v^2 u_{bH^b})\right]. \end{aligned}$$

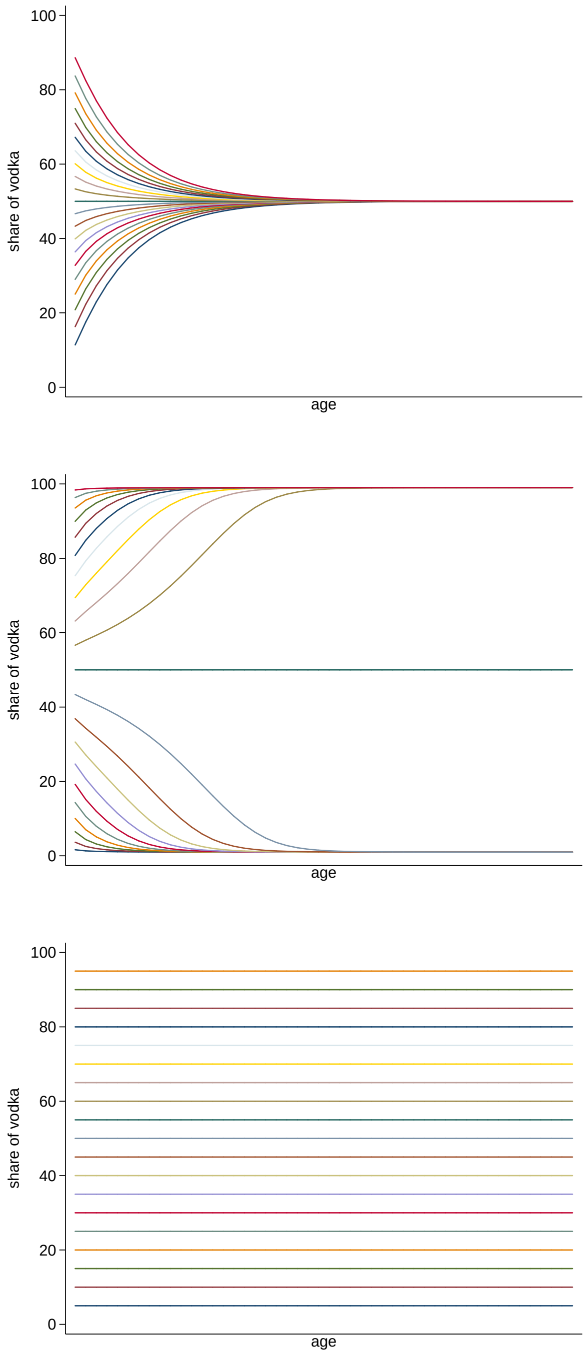
The terms in the first square brackets are all negative, while the terms in the second square brackets are all positive. Thus, depending on the relative magnitude of these terms, the first derivative can be positive or negative. The following utility specifications provide two examples, one with a unique and stable steady state and one with three steady states, two of which are stable and one is unstable. We again set $p_v = y = 1$ so that the consumption levels correspond to shares, and for simplicity we assume that $\beta = 1$ and $\delta = 0.5$. Then the utility parametrization $u = \sqrt{g} + \sqrt{H^g} + gH^g$ results in a one equilibrium, while $u = \sqrt{g} + \sqrt{H^g} + 5gH^g$ yields three equilibria.

References

Becker, Gary S. and Kevin M. Murphy, "A Theory of Rational Addiction," *Journal of Political Economy*, 1988, *96* (4), 675–700.

Gruber, Jonathan and Botond Köszegi, "Is Addiction "Rational"? Theory and Evidence," *Quarterly Journal of Economics*, 2001, *116* (4), 1261–1303.

Figure 1: Potential Number of Steady States in the 2-Good Becker-Murphy Model



Notes: These figures show the dynamic behavior of the share of vodka in the two-good habit formation model, starting from different initial conditions, i.e., different initial consumption shares. The three figures correspond to the three parametrizations specified in the text. The top panel has one stable steady state, the middle panel has three steady states, two stable and one unstable, and the bottom panel has an infinite number of steady states.