

Online Appendix of “Shopping for Lower Sales Tax Rates”

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A. Pricing Data

A.1 Nielsen Retail Scanner (Retail Prices)

With the Nielsen Retail Scanner Panel (NRP), price and quantity information is available at the store level for each UPC carried by a covered retailer and span the years 2006-2014. An average (quantity weighted) price is reported, by UPC, for each store every week.²⁴ NRP covers 125 product groups with more than 3.2 million individual UPCs. Units are consistently standardized and most products are measured in ounces (OZ, 51%), count (CT, 45%) or ml (ML, 2%).

A.2 PromoData (Wholesale Prices)

We use PromoData to measure wholesale prices for grocery and retail goods. Promo obtains its information from one (confidential) major wholesaler in each market.²⁵ One downside to this approach is that, since no single wholesaler carries every SKU in a given market, information about the universe of goods is not observed. Overall, Promo prices are available for 32 markets after removing redundant markets and combining overlapping markets.²⁶

Data on wholesale prices are available from 2006 - 2012. However, during 2012 the data loses a significant amount of coverage. For this reason, we perform robustness tests excluding 2012 data from our sample. PromoData contains all changes in price or deals that are run by the wholesaler. Thus, we take prices as constant between observations, based on the last observed price data. We then are able to collapse prices to a monthly level for each product group. To arrive at consistent unit prices within type of product (eg. product groups), we scale the observed wholesale prices by the number of goods in a ‘pack’ and by the size of the unit (eg. number of ounces in a candy bar and number of candy bars in a box). To make meaningful unit price comparisons we need to know the units associated with each good. Unfortunately unit information is often not provided

²⁴For a given store, coverage over time is stable and relatively complete across all years. Unit prices are calculated as $price/(prmult \times size1_amt)$.

²⁵By only using one wholesaler, Promo relies on the Robinson–Patman Anti-Price Discrimination Act of 1936 that prohibits price discrimination. In particular, it prevents wholesalers from offering special discounts to large chain stores but not to other, smaller retailers.

²⁶Leveraging this regional information provides additional variation but introduces more measurement error given less complete coverage in any given market both with respect to corresponding Nielsen product groups in the cross-section and time-series coverage of specific products.

or is inconsistently coded (e.g., CT, PACK, EACH, OZ, O etc.). We use the modal unit within UPC to impute missing values. The intuition is that if a product is recorded as being measured in OZ most of the time units are reported, it is probably measured in OZ.

A.3 Matching Wholesaler and Retailer Data

Given the large number of products in the retailer dataset we aggregate retail unit prices to the product group level before matching with wholesale prices. We assign products in the wholesaler data to Nielsen product groups by matching at the UPC level. The mapping is not one-to-one due to differences in end-digits when shifting to UPCs of different levels of granularity (e.g., some are reported with retailer specific end-digits, etc.). This leads to multiple Nielsen UPCs corresponding to a single Promo UPC for some goods. However, this appears to have little effect when merging Nielsen product groups to their Promo equivalents.

As a consistency check we also match retail and wholesale prices by UPC at a single point in time. The implied markup distribution supports the accuracy of both the raw data and our unit price calculations, with 90% of markups falling between -7% and 135%. We calculate Promo coverage of Nielsen product groups as the percentage of UPCs in each Nielsen product group that can be found in Promo. Overall, we see that about 4% of overall UPCs in Nielsen are also covered directly in the wholesale data for a given market. Aggregating across markets to the national level, this coverage increases somewhat.

The two datasets are merged based on the weekly date. That is, Promo prices are those associated with the week containing the Nielsen week-ending Saturday. For a Nielsen retailer using a 7-day period ending on Saturday the periods correspond closely. However, as mentioned above this is not the case for all retailers. For a retailer using a Thursday to Wednesday week, the Nielsen prices would pre-date the Promo prices by a few days.

Comparing unit prices is not completely straightforward as Promo units are missing for many products. As discussed above, we impute some missing units based on the modal unit reported in Promo for that UPC. When merging, we retain only UPCs for which the imputed Promo unit matches the Nielsen unit. A coarse attempt is made to standardize the more common Promo units before matching. In particular we assume *O* and *Z* refers to *OZ* and *C*, *CNT*, *PK*, *EA*, *EACH*, *STK*, *ROL*, *RL*, *PC*, *#*, *CTN* refer to *CT*.

B. Price and Quantity Response

While the majority of the paper discusses household responses in terms of changes in dollars of retail spending, it is possible that this may portray an incomplete view of household behavior. If retailers adjust prices or households shift spending to different types of goods, we may have a different interpretation of how the pre-tax spending response relates to actual consumption.

Appendix Table A.1 examines two other important margins. Columns 1 to 4 mirror the analysis done in Table 1 but using log-changes of quantities (items) purchased as the dependent

variable rather than log-changes in spending. We find qualitatively similar effects, with declines in quantities mirroring the declines in spending following an increase in sales tax rates. This indicates that households are likely not simply substituting lower quality and lower priced goods to reduce pre-tax spending.

Columns 5 to 8 test another potential confounding margin of adjustment. If retailers fully offset sales taxes, we might observe a decline in pre-tax spending with no actual decline in total (tax-inclusive) spending. Using data on both retail and wholesale prices we find that there is limited amounts of offsetting behavior on the part of firms. Retail prices decline by 0.15%-0.20% in the month following a 1% increase in sales tax rates, while wholesale prices remain unaffected.

The non-response of the wholesale prices may be driven by the fact that wholesalers are less geographically concentrated and so do not price to local conditions to the extent that retailers do. Another reason may be that wholesale prices tend to be more stable and feature fewer short-term sales than do retailers, leading to somewhat higher menu costs and a reduction in desire to change prices at high frequency. We leave a more complete analysis of the firm response to sales tax changes, taking into account the response of demand documented in this paper, to future research.

C. Derivation of the Shopping Model

C.1 Supporting Calculations

C.1.1 Within-Period Value Function, $U(C_{t_n}, \Delta t_n)$

Define $f(\Delta t; \alpha) = \frac{e^{\alpha \Delta t} - 1}{\alpha}$ with $f' = e^{\alpha \Delta t}$, $f^{(n)} = \alpha^{n-1} e^{\alpha \Delta t}$, $f(0) = 0$, $\lim_{\alpha \rightarrow 0} f(\Delta t; \alpha) = \Delta t$ and a second-order approximation around $\Delta t = 0$ is $f(\Delta t; \alpha) \approx (1 + \frac{\alpha}{2} \Delta t) \Delta t$. The Lagrangian of (3) is $\int_{x=0}^{\Delta t_n} [e^{-\rho t} u(C(t_n + x)) - \lambda e^{\delta x} C(t_n + x)] dx + \lambda S_{t_n}$. Defining $F(C, C', t) = e^{-\rho t} u(C(t_n + t)) - \lambda e^{\delta t} C(t_n + t)$, the general form of the Euler condition of this problem is $F_C = \frac{dF_{C'}}{dt} = F_{C'C} C' + F_{C'C'} + F_{C't} C''$. Since $F_{C'} = 0$, this reduces to $F_C = 0$, which implies $e^{\delta x} \lambda = e^{-\rho x} u'(C(t_n + x)) = e^{-\rho x} C(t_n + x)^{-1/\sigma}$. Hence $C(t_n + x) = \lambda^{-\sigma} e^{\gamma x}$ and $C_{t_n} = C(t_n) = \lambda^{-\sigma}$, where $\gamma = -(\delta + \rho)\sigma$,

$$C(t_n + x) = C_{t_n} e^{\gamma x}.$$

Plugging into the constraint yields $S_{t_n} = \int_0^{\Delta t_n} e^{\delta x} C(t_n + x) dx = \lambda^{-\sigma} \int_{x=0}^{\Delta t_n} e^{(\delta + \gamma)x} dx = \lambda^{-\sigma} f(\Delta t_n; \phi)$, with $\phi = \delta + \gamma = \delta - \sigma(\delta + \rho)$, so that

$$S_{t_n} = C_{t_n} \cdot f(\Delta t_n; \phi).$$

Plugging into the objective function and integrating yields²⁷

$$\begin{aligned}
U(C_{t_n}, \Delta t_n) &= \int_{x=0}^{\Delta t_n} e^{-\rho x} u(C(t_n + x)) dx = u(C_{t_n}) \int_{x=0}^{\Delta t_n} e^{-\rho x} e^{\frac{\sigma-1}{\sigma} \gamma x} dx \\
&= u(C_{t_n}) \int_{x=0}^{\Delta t_n} e^{\phi x} dx \\
&= u(C_{t_n}) \cdot f(\Delta t_n; \phi).
\end{aligned}$$

C.1.2 Inventories, S_{t_n}, s_{i,t_n}

Between shopping transactions, inventory evolves according to the first-order ordinary differential equation $\dot{s}_i(x) = -\delta s_i(x) - c_i(x)$, with boundary conditions $s_i(t_n) = s_{i,t_n}$ and $s_i(t_{n+1}^-) = 0$. The solution for $x \in [t_n, t_{n+1})$ is

$$s_i(t_n + x) = e^{-\delta x} \left[s_{i,t_n} - \int_{z=0}^x e^{\delta z} c_i(t_n + z) dz \right] \quad (14)$$

Hicksian demand $c_i(t)$ is a function of the relative price at the transaction date t_n , $p_{i,t_n}/P_{t_n}$ such that $c_i(t_n + z) = b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} C(t_n + z) = b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} C_{t_n} e^{\gamma z}$. We can use individual inventories $s_i(t_n)$ to define inventories of the composite consumption good

$$\begin{aligned}
S(t_n + x) &\equiv \frac{\sum_i p_{i,t_n} s_i(t_n + x)}{P_{t_n}} = e^{-\delta x} \left[S_{t_n} - \int_{z=0}^x e^{\delta z} \underbrace{\frac{\sum_i b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} p_{i,t_n}}{P_{t_n}}}_{=1} C(t_n + z) dz \right] \\
&= e^{-\delta x} \left[S_{t_n} - C_{t_n} \int_{z=0}^x e^{(\delta+\gamma)z} dz \right] \\
&= e^{-\delta x} \left[S_{t_n} - C_{t_n} f(x; \phi) \right].
\end{aligned}$$

The condition $s_i(t_n + \Delta t_n) = s_i(t_{n+1}^-) = 0$ implies $S(t_n + \Delta t_n) = S(t_{n+1}^-) = 0$ and

$$S_{t_n} = C_{t_n} f(\Delta t_n; \phi).$$

Similarly, using $s_i(t_{n+1}^-) = 0 = e^{-\delta \Delta t_n} \left[s_{i,t_n} - \int_{z=0}^{\Delta t_n} e^{\delta z} c_i(t_n + z) dz \right]$, beginning-of-period inventories for the individual goods are

$$\begin{aligned}
s_{i,t_n} &= \int_{z=0}^{\Delta t_n} e^{\delta z} c_i(t_n + z) dz \\
&= b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} C_{t_n} \int_{z=0}^{\Delta t_n} e^{(\delta+\gamma)z} dz = b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} C_{t_n} f(\Delta t_n; \phi) \\
&= b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} S_{t_n}
\end{aligned}$$

²⁷ Also note that $U(C_{t_n}, \Delta t_n) = U(S_{t_n}, \Delta t_n) = u(S_{t_n}) \cdot f(\Delta t_n; \phi)^{1/\sigma}$.

and the expenditure share of good i is

$$B_{i,t_n} = \frac{p_{i,t_n} S_{i,t_n}}{P_{t_n} S_{t_n}} = b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{1-\eta}.$$

C.1.3 Tax Elasticity of the Price Index

The effective cost-of-living price index is $P(\tau) = [b_\tau(1+\tau)^{1-\eta}\tilde{p}_\tau^{1-\eta} + b_e\tilde{p}_e^{1-\eta}]^{1/(1-\eta)}$, where \tilde{p}_i is the pre-tax price so that $p_\tau = (1+\tau)\tilde{p}_\tau$ and $p_e = \tilde{p}_e$. Hence

$$\begin{aligned} \frac{d \ln P(\tau)}{d \ln(1+\tau)} &= \frac{1+\tau}{P} \frac{dP}{d(1+\tau)} \\ &= \frac{1+\tau}{P} \frac{1}{1-\eta} P^\eta (1-\eta)(1+\tau)^{-\eta} b_\tau \tilde{p}_\tau^{1-\eta} \\ &= b_\tau \left(\frac{(1+\tau)\tilde{p}_\tau}{P} \right)^{1-\eta} = b_\tau \left(\frac{p_\tau}{P} \right)^{1-\eta}. \end{aligned}$$

The taxable expenditure share is

$$B_\tau = \frac{p_\tau s_\tau}{PS} = \frac{p_\tau}{PS} b_\tau \left(\frac{p_\tau}{P} \right)^{-\eta} S = b_\tau \left(\frac{p_\tau}{P} \right)^{1-\eta}.$$

Hence,

$$\frac{d \ln P(\tau)}{d \ln(1+\tau)} = B_\tau.$$

C.2 Model Solution

C.2.1 Transaction Interval, Δt_n

Define $C = C_{\Delta t}$ with consumption flow $C_{\Delta t} = \int_{x=0}^{\Delta t} C(t_n + x) dx = C_{t_n} f(\gamma)$ so that $S = C \frac{f(\phi)}{f(\gamma)}$ and

$$\begin{aligned} U &= u(C) \cdot f(\gamma)^{\frac{1}{\sigma}-1} \cdot f(\phi) = u(C/f(\gamma)) \cdot f(\phi) = u(S) \cdot f(\phi)^{\frac{1}{\sigma}} \\ K &= \kappa + PC \cdot \frac{f(\phi)}{f(\gamma)} = \kappa + PS. \end{aligned}$$

The partial derivatives of U and K with respect to C are

$$\begin{aligned} \partial_C K' &= P \frac{f(\phi)}{f(\gamma)} = \frac{PS}{C} \\ \partial_C U' &= u'(C) \cdot f(\gamma)^{\frac{1}{\sigma}-1} f(\phi) = U \cdot \frac{u'(C)}{u(C)} \end{aligned}$$

such that (6) becomes

$$V' = \frac{\partial_C U'}{\partial_C K'} = \frac{(1 - \frac{1}{\sigma})U}{PS}.$$

The partial derivatives of U and K with respect to Δt are

$$\partial_{\Delta t} K' = PC[-f(\gamma)^{-2}f(\phi)e^{\gamma\Delta t} + f(\gamma)^{-1}e^{\phi\Delta t}] = PS\left[\frac{e^{\phi\Delta t}}{f(\phi)} - \frac{e^{\gamma\Delta t}}{f(\gamma)}\right]$$

and

$$\begin{aligned}\partial_{\Delta t} U' &= u(C)\left[\left(\frac{1}{\sigma} - 1\right)f(\gamma)^{\frac{1}{\sigma}-2}f(\phi)e^{\gamma\Delta t} + f(\gamma)^{\frac{1}{\sigma}-1}e^{\phi\Delta t}\right] \\ &= u(C)f(\gamma)^{\frac{1}{\sigma}-1}f(\phi)\left[\frac{e^{\phi\Delta t}}{f(\phi)} - \left(1 - \frac{1}{\sigma}\right)\frac{e^{\gamma\Delta t}}{f(\gamma)}\right] \\ &= U\left[\frac{e^{\phi\Delta t}}{f(\phi)} - \frac{e^{\gamma\Delta t}}{f(\gamma)} + \frac{1}{\sigma}\frac{e^{\gamma\Delta t}}{f(\gamma)}\right] = U\left[\frac{\partial_{\Delta t} K}{PS} + \frac{1}{\sigma}\frac{e^{\gamma\Delta t}}{f(\gamma)}\right].\end{aligned}$$

Necessary condition for Δt_n Necessary condition (8) can also be written as

$$\partial_{\Delta t} U'_{t_n} - e^{-\rho\Delta t_n}\rho V_{t_{n+1}} = [\partial_{\Delta t} K'_{t_n} - e^{-r\Delta t_n}r w_{t_{n+1}}]e^{(r-\rho)\Delta t_n} V'_{t_{n+1}}.$$

The right-hand side is

$$\begin{aligned}e^{(r-\rho)\Delta t_n} V'(w_{t_{n+1}})[\partial_{\Delta t} K'_{t_n} - r(w_{t_n} - K_{t_n})] &= e^{(r-\rho)\Delta t_n} V'(w_{t_{n+1}})[\partial_{\Delta t} K'_{t_n} - e^{-r\Delta t_n}r w_{t_{n+1}}] \\ &= \left(1 - \frac{1}{\sigma}\right)U_{t_n}\left[\frac{\partial_{\Delta t} K'_{t_n}}{P_{t_n}S_{t_n}} - \frac{e^{-r\Delta t_n}r w_{t_{n+1}}}{P_{t_n}S_{t_n}}\right] \\ &= \left(1 - \frac{1}{\sigma}\right)U_{t_n}\left[\frac{e^{\phi\Delta t_n}}{f(\phi)} - \frac{e^{\gamma\Delta t_n}}{f(\gamma)} - \frac{e^{-r\Delta t_n}r w_{t_{n+1}}}{P_{t_n}S_{t_n}}\right]\end{aligned}$$

and the left-hand side is

$$\begin{aligned}\partial_{\Delta t} U'_{t_n} - e^{-\rho\Delta t_n}\rho V(w_{t_{n+1}}) &= U_{t_n}\left[\frac{\partial_{\Delta t} K'_{t_n}}{P_{t_n}S_{t_n}} + \frac{1}{\sigma}\frac{e^{\gamma\Delta t_n}}{f(\gamma)}\right] - e^{-\rho\Delta t_n}\rho V(w_{t_{n+1}}) \\ &= U_{t_n}\left[\frac{e^{\phi\Delta t_n}}{f(\phi)} - \left(1 - \frac{1}{\sigma}\right)\frac{e^{\gamma\Delta t_n}}{f(\gamma)}\right] - e^{-\rho\Delta t_n}\rho V(w_{t_{n+1}}).\end{aligned}$$

Hence, necessary condition (8), which implicitly defines Δt_n , can be rewritten as

$$\frac{\rho e^{-\rho\Delta t_n} V(w_{t_{n+1}})}{U(S_{t_n}, \Delta t_n)} - \left(1 - \frac{1}{\sigma}\right)\frac{r e^{-r\Delta t_n} w_{t_{n+1}}}{P_{t_n} S_{t_n}} = \frac{1}{\sigma}\frac{e^{\phi\Delta t_n}}{f(\Delta t_n; \phi)} \quad (15)$$

or substituting out inventories,

$$\frac{\rho e^{-\rho\Delta t_n} V(w_{t_{n+1}})}{u(C_{t_n})} - \left(1 - \frac{1}{\sigma}\right)\frac{r e^{-r\Delta t_n} w_{t_{n+1}}}{P_{t_n} C_{t_n}} = \frac{1}{\sigma}e^{\phi\Delta t_n}.$$

C.2.2 Final Stationary State (starting at t_{ss})

In the stationary state with $r = \rho$, (4) implies

$$V_{t_{ss}} = (1 - e^{-\rho\Delta t_{ss}})^{-1} U_{t_{ss}} \quad (16)$$

$$w_{t_{ss}} = (1 - e^{-r\Delta t_{ss}})^{-1} K_{t_{ss}} = (1 - e^{-r\Delta t_{ss}})^{-1} (\kappa + P_{t_{ss}} S_{t_{ss}}) \quad (17)$$

Plugging the stationary-state value function and wealth into (15) and evaluating at the stationary state $\rho = r$, noting that $e^{-r\Delta t} r (1 - e^{-r\Delta t})^{-1} = f(\Delta t; r)^{-1}$, yields (9),

$$(1 - \sigma) \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} = e^{\phi\Delta t_{ss}} \frac{f(\Delta t_{ss}; r)}{f(\Delta t_{ss}; \phi)} - 1$$

or in terms of consumption,

$$(1 - \sigma) \frac{\kappa}{P_{t_{ss}} C_{t_{ss}}} = e^{\phi\Delta t_{ss}} f(\Delta t_{ss}; r) - f(\Delta t_{ss}; \phi). \quad (18)$$

Furthermore, by plugging (17) into (9), we can express the optimal shopping cycle in the stationary state instead as a function of the total level of wealth in stationary state, $w_{t_{ss}}$,²⁸

$$(1 - \sigma) \left[\frac{\kappa}{(1 - e^{-r\Delta t_{ss}}) w_{t_{ss}} - \kappa} \right] = e^{\phi\Delta t_{ss}} \frac{f(\Delta t_{ss}; r)}{f(\Delta t_{ss}; \phi)} - 1. \quad (19)$$

Approximate steady-state trip interval (“square-root formula”) Define the right-hand side of (18)

$$F(\Delta t) = e^{\phi\Delta t} f(\Delta t; r) - f(\Delta t; \phi).$$

Taking a second-order Taylor expansion of F around $\Delta t = 0$

$$F(\Delta t) \approx F(0) + F'(0)\Delta t + F''(0) \frac{(\Delta t)^2}{2},$$

noting that

$$\begin{aligned} F(0) &= 0 \\ F'(\Delta t) &= \phi e^{\phi\Delta t} f(\Delta t; \phi) + e^{(\phi+\delta)\Delta t} - e^{\phi\Delta t} && \Rightarrow F'(0) = 0 \\ F''(\Delta t) &= \phi^2 e^{\phi\Delta t} f(\Delta t; \phi) + (2\phi + r) e^{(\phi+r)\Delta t} - \phi e^{\phi\Delta t} && \Rightarrow F''(0) = (1 - \sigma)(\delta + r) \end{aligned}$$

²⁸ Note that if $\sigma = 1$ (i.e., income effect equals substitution effect) then Δt_{ss} is not defined by (9) since the LHS=RHS=0 independent of Δt_{ss} , but instead is pinned down by the steady-state budget constraint.

yields

$$F(\Delta t) \approx (1 - \sigma)(\delta + r) \frac{(\Delta t)^2}{2}.$$

If $\sigma = 0$ (which we cannot reject based on our estimates of the long-run spending response), then consumption is constant, in particular $C_{t_n} = C$, and does not depend on Δt . Hence the left-hand side of (18) is not affected by the Taylor expansion around $\Delta t = 0$. Therefore, substituting the approximation of the right-hand side into (18) yields the approximate square root formula in the text,

$$\Delta t_{ss} \approx \sqrt{\frac{\kappa}{\frac{\delta+r}{2} P_{t_{ss}} C_{t_{ss}}}}.$$

C.2.3 Interim Shopping Period (starting at t_{ss-1})

A. Change of the interim-period interval (Δt_{ss-1}) Using (15) we have

$$\begin{aligned} \frac{1}{\sigma} \frac{e^{\phi \Delta t_{ss-1}}}{f(\Delta t_{ss-1}; \phi)} &= \frac{\rho e^{-\rho \Delta t_{ss-1}} V(w_{t_{ss}})}{U(C_{t_n}, \Delta t_n)} - \left(1 - \frac{1}{\sigma}\right) \frac{r e^{-r \Delta t_{ss-1}} w_{t_{ss}}}{P_{t_{ss-1}} S_{t_{ss-1}}} \\ &= \frac{\rho e^{-\rho \Delta t_{ss-1}} (1 - e^{-\rho \Delta t_{ss}})^{-1} U_{t_{ss}}}{U_{t_{ss-1}}} - \left(1 - \frac{1}{\sigma}\right) \frac{r e^{-r \Delta t_{ss-1}} (1 - e^{-r \Delta t_{ss}})^{-1} (\kappa + P_{t_{ss}} S_{t_{ss}})}{P_{t_{ss-1}} S_{t_{ss-1}}} \\ &= r e^{-r \Delta t_{ss-1}} (1 - e^{-r \Delta t_{ss}})^{-1} \left[\frac{U_{t_{ss}}}{U_{t_{ss-1}}} - \left(1 - \frac{1}{\sigma}\right) \left(\frac{P_{t_{ss}} S_{t_{ss}}}{P_{t_{ss-1}} S_{t_{ss-1}}} + \frac{\kappa}{P_{t_{ss-1}} S_{t_{ss-1}}} \right) \right]. \end{aligned}$$

Using (7) we find an expression for $U_{t_{ss}}/U_{t_{ss-1}}$,

$$\begin{aligned} \frac{U_{t_{ss}}}{U_{t_{ss-1}}} &= \frac{u(S_{t_{ss}}) f(\Delta t_{ss}; \phi)^{1/\sigma}}{u(S_{t_{ss-1}}) f(\Delta t_{ss-1}; \phi)^{1/\sigma}} = \left(\frac{S_{t_{ss}}}{S_{t_{ss-1}}} \right)^{1-1/\sigma} \left(\frac{f(\Delta t_{ss}; \phi)}{f(\Delta t_{ss-1}; \phi)} \right)^{1/\sigma} \\ &= e^{(\sigma-1)(r-\rho)\Delta t_{ss-1}} \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{1-\sigma} \frac{f(\Delta t_{ss}; \phi)}{f(\Delta t_{ss-1}; \phi)} \end{aligned}$$

and

$$\begin{aligned} \frac{P_{t_{ss}} S_{t_{ss}}}{P_{t_{ss-1}} S_{t_{ss-1}}} + \frac{\kappa}{P_{t_{ss-1}} S_{t_{ss-1}}} &= \frac{P_{t_{ss}} S_{t_{ss}}}{P_{t_{ss-1}} S_{t_{ss-1}}} \left(1 + \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} \right) \\ &= e^{\sigma(r-\rho)\Delta t_{ss-1}} \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{1-\sigma} \frac{f(\Delta t_{ss}; \phi)}{f(\Delta t_{ss-1}; \phi)} \left(1 + \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} \right). \end{aligned}$$

Plugging back in and evaluating at $\rho = r$,

$$\begin{aligned} \frac{1}{\sigma} \frac{e^{\phi \Delta t_{ss-1}}}{f(\Delta t_{ss-1}; \phi)} &= e^{-r(\Delta t_{ss-1} - \Delta t_{ss})} f(\Delta t_{ss}; r)^{-1} \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{1-\sigma} \frac{f(\Delta t_{ss}; \phi)}{f(\Delta t_{ss-1}; \phi)} \left[1 - \left(1 - \frac{1}{\sigma}\right) \left(1 + \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} \right) \right] \\ &= e^{-r(\Delta t_{ss-1} - \Delta t_{ss})} f(\Delta t_{ss}; r)^{-1} \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{1-\sigma} \frac{f(\Delta t_{ss}; \phi)}{f(\Delta t_{ss-1}; \phi)} \frac{1}{\sigma} \left[1 + (1 - \sigma) \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} \right]. \end{aligned}$$

Therefore,

$$(1 - \sigma) \frac{\kappa}{P_{t_{ss}} S_{t_{ss}}} = e^{\phi \Delta t_{ss-1} + r(\Delta t_{ss-1} - \Delta t_{ss})} \frac{f(\Delta t_{ss}; r)}{f(\Delta t_{ss}; \phi)} \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{-(1-\sigma)} - 1.$$

Substituting the left-hand side with (9)

$$\left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{(1-\sigma)} = e^{(\phi+r)(\Delta t_{ss-1} - \Delta t_{ss})} = e^{(1-\sigma)(\delta+r)(\Delta t_{ss-1} - \Delta t_{ss})}$$

and taking logs yields

$$\Delta t_{ss-1} - \Delta t_{ss} = \frac{\ln(P_{t_{ss}}/P_{t_{ss-1}})}{\delta + r}.$$

Elasticity Hence, we obtain (11) by the following approximation,

$$\frac{\ln(P_{t_{ss}}/P_{t_{ss-1}})}{(\delta + r)\Delta t_{ss}} = \frac{\Delta t_{ss-1} - \Delta t_{ss}}{\Delta t_{ss}} \approx -\ln(\Delta t_{ss}/\Delta t_{ss-1})$$

such that

$$\begin{aligned} \varepsilon_{\Delta t_{ss-1}} &= \frac{d \ln(\Delta t_{ss}/\Delta t_{ss-1})}{d \ln(1 + \tau_{t_{ss}})} \Big|_{\Delta t_{ss} \text{ cons}} = -\frac{1}{(\delta + r)\Delta t_{ss}} \frac{d \ln(P_{t_{ss}}/P_{t_{ss-1}})}{d \ln(1 + \tau_{t_{ss}})} \\ &= -\frac{B_\tau}{(\delta + r)\Delta t_{ss}}. \end{aligned}$$

B. Change of iterim-period spending ($s_{i,t_{ss-1}}$) Beginning-of-period inventory of good i is

$$s_{i,t_n} = b_i \left(\frac{p_{i,t_n}}{P_{t_n}} \right)^{-\eta} S_{t_n}$$

such that

$$\frac{s_{i,t_{ss-1}}}{s_{i,t_{ss}}} = \left(\frac{p_{i,t_{ss}}}{p_{i,t_{ss-1}}} \right)^\eta \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{-\eta} \frac{S_{t_{ss-1}}}{S_{t_{ss}}}.$$

Substituting Euler equation (7) evaluated at $\rho = r$ yields

$$\frac{s_{i,t_{ss-1}}}{s_{i,t_{ss}}} = \left(\frac{p_{i,t_{ss}}}{p_{i,t_{ss-1}}} \right)^\eta \left(\frac{P_{t_{ss}}}{P_{t_{ss-1}}} \right)^{\sigma-\eta} \frac{f(\Delta t_{ss-1}; \phi)}{f(\Delta t_{ss}; \phi)}.$$

Using the fact that $\frac{d \ln(P_{t_{ss}}/P_{t_{ss-1}})}{d \ln(1 + \tau_{t_{ss}})} = B_\tau$, the compensated short-run spending elasticity of a forward-looking consumer is

$$\varepsilon_{s_{i,t_{ss-1}}} \equiv \frac{d \ln(s_{i,t_{ss}}/s_{i,t_{ss-1}})}{d \ln(1 + \tau_{t_{ss}})}$$

$$\begin{aligned}
&= -(\sigma - \eta) \frac{d \ln(P_{t_{ss}}/P_{t_{ss-1}})}{d \ln(1 + \tau_{t_{ss}})} - \eta \cdot \frac{d \ln(p_{i,t_{ss}}/p_{i,t_{ss-1}})}{d \ln(1 + \tau_{t_{ss}})} - \frac{d \ln(f(\Delta t_{ss-1}; \phi)/f(\Delta t_{ss}; \phi))}{d \ln(1 + \tau_{t_{ss}})} \\
&= -(\sigma - \eta) B_\tau - \eta \cdot 1_{\{i=\tau\}} - \frac{d \ln(f(\Delta t_{ss-1}; \phi)/f(\Delta t_{ss}; \phi))}{d \ln(1 + \tau_{t_{ss}})} \\
&= \varepsilon_i^c - \frac{d \ln(f(\Delta t_{ss-1}; \phi)/f(\Delta t_{ss}; \phi))}{d \ln(1 + \tau_{t_{ss}})}.
\end{aligned}$$

Hence, the additional sensitivity of spending relative to consumption is driven by the last term.

Elasticity Evaluating the derivatives of f around $d\tau = 0$ such that $d \ln f(\Delta t_{ss-1}; \phi) \approx d \ln f(\Delta t_{ss}; \phi)$ and using (11) we get

$$\begin{aligned}
\frac{d \ln f(\Delta t_{ss-1}; \phi)}{d \ln(1 + \tau_{t_{ss}})} - \frac{d \ln f(\Delta t_{ss}; \phi)}{d \ln(1 + \tau_{t_{ss}})} &\approx \frac{e^{\phi \Delta t_{ss}}}{f(\Delta t_{ss}; \phi)} \frac{d(\Delta t_{ss-1} - \Delta t_{ss})}{d \ln(1 + \tau_{t_{ss}})} \\
&= \frac{e^{\phi \Delta t_{ss}}}{f(\Delta t_{ss}; \phi)} \frac{B_\tau}{\delta + r}.
\end{aligned}$$

Taking a first order approximation of $G(\phi) \equiv \frac{e^{\phi \Delta t}}{f(\Delta t; \phi)} = \frac{\phi e^{\phi \Delta t}}{e^{\phi \Delta t} - 1}$ around $\phi = 0$, $G(\phi) \approx \frac{1}{\Delta t} + \frac{1}{2}\phi$, yields

$$\frac{d \ln(f(\Delta t_{ss-1}; \phi)/f(\Delta t_{ss}; \phi))}{d \ln(1 + \tau_{t_{ss}})} \approx \frac{B_\tau}{\delta + r} \left(\frac{1}{\Delta t_{ss}} + \frac{\phi}{2} \right).$$

Evaluating G at $\phi = 0$ instead yields the approximation in (12).

Proof: Using de l'Hopital's rule, $G(0) = \lim_{\phi \rightarrow 0} G(\phi) = \frac{1}{\Delta t}$. After some algebra, the derivative of G simplifies to $G'(\phi) = \frac{e^{\phi \Delta t}(e^{\phi \Delta t} - 1 - \phi \Delta t)}{(e^{\phi \Delta t} - 1)^2}$. Using de l'Hopital's rule again, $G'(0) = \lim_{\phi \rightarrow 0} G'(\phi) = \frac{1}{2}$.

C.2.4 Pre Tax Change Periods (until t_{ss-1})

Consider the problem of choosing how to space N trips planned to occur before the interim shopping trip at $t_{ss-1} = t_\tau^-$.²⁹ Without much loss of generality we start model time at a date that corresponds to a shopping transaction. The goal is to show that for an appropriate choice of tax change date t_τ there is a solution involving a constant trip interval $\Delta t = \Delta t_{ss-2} = \Delta t_{ss-q} \forall q \geq 2$ and constant beginning-of-period consumption $C = C_{t_{ss-2}} = C_{t_{ss-q}} \forall q \geq 2$. Define the start and end dates of the pre tax change period

$$\begin{aligned}
t_0 &= 0 \\
t_N &= t_\tau^- = t_{ss-1}.
\end{aligned}$$

²⁹Note that it is best for the household to take the interim trip as close to t_τ as possible, all else constant.

There are $N + 1$ transaction dates and N transaction intervals. Also define $V(w_{t_\tau^-})$ as the value of the problem starting from the interim shopping trip at t_τ^- given accumulated wealth $w_{t_\tau^-}$. The problem is

$$V(w_0) = \max_{w_{t_\tau^-}, \Delta t_0, \dots, \Delta t_{N-1}, C_{t_0}, \dots, C_{t_{N-1}}} \sum_{k=0}^{N-1} e^{-\rho \sum_{j=0}^{k-1} \Delta t_j} U(C_{t_k}, \Delta t_k) + e^{-\rho t_\tau^-} V(w_{t_\tau^-})$$

subject to

$$\begin{aligned} t_\tau^- &= \sum_{k=0}^{N-1} \Delta t_k \\ w_0 &= \sum_{k=0}^{N-1} e^{-r \sum_{j=0}^{k-1} \Delta t_j} K_{t_k} + e^{-rt_\tau^-} w_{t_\tau^-} \end{aligned}$$

where the multiplier on first constraint is λ_1 and on second constraint is λ_2 .

Necessary condition for Δt_n

$$e^{-\rho t_n} \lambda_1 = \left[\partial_\Delta U'_{t_n} - \rho \sum_{k=n+1}^{N-1} e^{-r \sum_{j=0}^{k-1} \Delta t_j} U_{t_k} \right] - \lambda_2 \left[\partial_\Delta K'_{t_n} - r \sum_{k=n+1}^{N-1} e^{-\rho \sum_{j=0}^{k-1} \Delta t_j} K_{t_k} \right]. \quad (20)$$

Necessary condition for C_{t_n}

$$\partial_C U'_{t_n} = \lambda_2 \cdot \partial_C K'_{t_n}. \quad (21)$$

Necessary condition for $w_{t_\tau^-}$

$$e^{-\rho t_\tau^-} V'(w_{t_\tau^-}) = \lambda_2 e^{-rt_\tau^-} \quad (22)$$

Using (22) and $r = \rho$ we get

$$\lambda_2 = V'(w_{t_\tau^-})$$

The consumption Euler equation is

$$\frac{\partial_C U'_{t_n}}{\partial_C U'_{t_{n+1}}} = \frac{P_{t_n}}{P_{t_{n+1}}} \frac{f(\Delta t_n; \phi)}{f(\Delta t_{n+1}; \phi)} \frac{f(\Delta t_n; \gamma)}{f(\Delta t_{n+1}; \gamma)}$$

The transaction Euler equation is obtained using (20),

$$\partial_\Delta U'_{t_n} - \rho e^{-\rho \Delta t_n} U'_{t_{n+1}} - [\partial_\Delta K'_{t_n} - r e^{-r \Delta t_n} K_{t_{n+1}}] V'(w_{t_\tau^-}) = e^{-\rho \Delta t_n} \partial_\Delta U'_{t_{n+1}} - e^{-r \Delta t_n} \partial_\Delta K'_{t_{n+1}} V'(w_{t_\tau^-}).$$

Using the constant guess for the solution, $\Delta t_n = \Delta t_{t_{ss-2}} = \Delta t$ and $C_{t_n} = C_{t_{ss-2}} = C$, we obtain a condition similar to the steady state equation for the post tax transaction interval,

$$\partial_\Delta U' - \rho \frac{e^{-\rho \Delta t}}{1 - e^{-\rho \Delta t}} U = [\partial_\Delta K' - r \frac{e^{-r \Delta t}}{1 - e^{-r \Delta t}} K] V'(w_{t_\tau^-}).$$

Using similar steps as in the derivation of the steady state above, we can combine this relationship with (21) to yield

$$(1 - \sigma) \frac{\kappa}{P_{t_{ss-2}} S_{t_{ss-2}}} = e^{\phi \Delta t_{ss-2}} \frac{f(\Delta t_{ss-2}; r)}{f(\Delta t_{ss-2}; \phi)} - 1. \quad (23)$$

Furthermore, since $V'(w_{t_{\tau}^-})$ is also the multiplier in the post-tax steady state, we can relate $C_{t_{ss-2}}$ and $C_{t_{ss}}$ through (21),

$$\partial_C U'_{t_n} = V'(w_{t_{\tau}^-}) \partial_C K'_{t_n},$$

such that

$$u'(C_{t_{ss-2}}) f(\Delta t_{ss-2}; \gamma)^{1/\sigma-1} f(\Delta t_{ss-2}; \phi) = V'(w_{t_{\tau}^-}) P_{t_{ss-2}} \frac{f(\Delta t_{ss-2}; \phi)}{f(\Delta t_{ss-2}; \gamma)}$$

which reduces to

$$u'(C_{t_{ss-2}}) = V'(w_{t_{\tau}^-}) P_{t_{ss-2}} f(\Delta t_{ss-2}; \gamma)^{-1/\sigma}.$$

Hence,

$$C_{t_{ss-2}} = \left(P_{t_{ss-2}} V'(w_{t_{\tau}^-}) \right)^{-\sigma}$$

and

$$C_{t_{ss}} = \left(P_{t_{ss}} V'(w_{t_{\tau}^-}) \right)^{-\sigma}$$

such that

$$\frac{C_{t_{ss-2}}}{C_{t_{ss}}} = \left(\frac{P_{t_{ss-2}}}{P_{t_{ss}}} \right)^{-\sigma}.$$

If we use $S_{t_{ss-2}} = C_{t_{ss-2}} f(\Delta t_{ss-2}; \phi)$ in (23), we have two equations in Δt_{ss-2} and $C_{t_{ss-2}}$, which we solve to get the pre tax change solution

$$(1 - \sigma) \frac{\kappa}{P_{t_{ss-2}} C_{t_{ss-2}} f(\Delta t_{ss-2}; \phi)} = e^{\phi \Delta t_{ss-2}} \frac{f(\Delta t_{ss-2}; r)}{f(\Delta t_{ss-2}; \phi)} - 1$$

$$\frac{C_{t_{ss-2}}}{C_{t_{ss}}} = \left(\frac{P_{t_{ss-2}}}{P_{t_{ss}}} \right)^{-\sigma}$$

To make sure $\sum_{k=0}^{N-1} \Delta t_k = t_{\tau}$ is satisfied we set

$$t_{\tau-} = N \cdot \Delta t_{ss-2}.$$

This solution has a straightforward interpretation: Intertemporal consumption allocation satisfies the standard consumption Euler equation (even in the presence of transaction fixed costs and product storability) and the optimal trip interval in the pre tax change period reflects the same trade-offs as in the final steady state. Figure 4 highlights these two features of optimal transaction intervals and spending and consumption plans.

Table A.1: Quantity and Price Response

Dependent variable:	A. Quantity Response				B. Price Response			
	$\Delta\ln(\text{taxable})$	$\Delta\ln(\text{exempt})$	$\Delta\ln(\text{taxable})$	$\Delta\ln(\text{exempt})$	$\Delta\ln(\text{retail price})$		$\Delta\ln(\text{wholesale price})$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta\ln(1 + \text{total sales tax rate})$	-2.330*** (0.479)	-1.458*** (0.458)			-0.215*** (0.036)		-0.008* (0.004)	
$\Delta\ln(1 + \text{state sales tax rate})$			-2.245** (0.908)	-1.744*** (0.566)		-0.171** (0.069)		-0.007 (0.015)
Period FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Household FE	Yes	Yes	Yes	Yes				
Product FE					Yes	Yes	Yes	Yes
ZIP3 FE					Yes	Yes	Yes	Yes
Observations	4,140,969	4,142,698	5,928,529	5,928,499	4,333,000	5,862,621	4,333,000	5,862,621
R-squared	0.014	0.014	0.013	0.013	0.011	0.010	0.189	0.177

Notes: Dependent variables in columns 1 to 4 are monthly changes in logged quantities (items) purchased by each household in the Nielsen Consumer Panel. Dependent variables in column 5 to 8 are monthly changes in sales-weighted average prices by product group and ZIP-3 code for all retailers in the Nielsen Retail Scanner Panel. For robustness, the dependent variables are winsorized at the 1% level. Regressions span 2004-2014 for state sales tax rate changes using the Nielsen Consumer Panel (columns 3 and 4) respectively 2006-2014 using the Nielsen Retail Scanner Panel (columns 6 and 8), and 2008-2014 for total sales tax rate changes (columns 1, 2, 5, and 7). Robust standard errors in parentheses adjust for arbitrary within-product correlations and heteroskedasticity and are clustered at the ZIP-3 code for total sales tax rate changes and at the state level for state sales tax rate changes.